obvious meaning is a yard cube. This obscures the point of a very good story in which mathematical and legal reasoning are shown to differ in their results. There are some curious stories of magic which would have repaid fuller investigation.

*Nature Pictures for Little People.* By W. Mawer, and others. (London: The Sunday School Association.

SOME parts of this book are very good; others impress us much less favourably. In order to test whether the little people for whom the book is intended were interested in its contents, we gave it to a few average boys to read, and found that their verdict was the same as that expressed above. The authors frequently assume that their young readers are familiar with things not commonly seen, and with expressions not usually found in children's reading-books. For instance, one section of the book is headed 'Απτέρυξ and begins with the sentence, "Rara avis in terris indeed!" This is very well in its way, but is out of place in a book of this character. Also, the numerous small witticisms and puns do not add to the interest or value of the descriptions. "You will wonder," we find in an account of whales "notwithstanding all you have read, how the big whale in the picture was drawn out of the sea and placed upon that rock. I have an opinion of my own upon the subject, and will confide it to you: the artist *drew* him up there." The following specimen is also unattractive, if not misleading. "When collecting those tiny shells which you find ready perforated for threading into necklaces, do you wonder *why* they are so perforated? If so, let me tell you. The little creature which lives in the shell is the favourite food of those bigger ones which have been introduced to you as Roaring Buckies—namely, whelks, as well as purples: and to get out the sweet morsel—oh! so sweet—with their tongues (or what you may call their tongues) they file out the little round hole, and then—oh!" There is much brilliant writing of this character, the style being after that in Kingsley's "Water Babies," and a very long way after. In our opinion, however, the best parts are those *not* containing composition of the kind quoted. Several excellent illustrations, and two or three simply-worded and interesting sections, are the book's only redeeming features.

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## LETTERS TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

### The Foundations of Dynamics.

DYNAMICAL investigations are often made to depend upon the following three propositions, viz., *the principle of linear momentum*, *the principle of angular momentum*, and *the principle of energy*. In treatises which are concerned with ordinary mechanical systems, such as rigid bodies, elastic solids and *frictionless* fluids, the word energy is generally employed in the restricted sense of mechanical energy, and is confined to two particular species, viz. kinetic energy arising from such motions of the system as can be controlled by ordinary mechanical agencies, and potential energy arising from the configuration of the system or from its position in space. All other forms of energy, such as molecular kinetic energy arising from the production of heat by friction, and chemical potential energy contained in fuel, explosive compounds, &c., are excluded from consideration. In systems of this kind the sum of the kinetic and potential energies is an invariable quantity, and this proposition may be termed the *principle of the conservation of mechanical energy*.

It is important to recollect that the principle of momentum is a proposition of a wider character than the principle of the conservation of mechanical energy, for the former proposition is true when the system possesses viscosity or internal friction which gives rise to a conversion of mechanical energy into heat. From this fact it follows that the principles of momentum and energy cannot be regarded as axiomatic, but depend upon certain propositions of a more fundamental character, which ought to be capable of explaining why it is that the principle of momentum is true in the case of viscous systems, whilst the principle of the conservation of mechanical energy does not hold good.

In order to examine this question we shall start with Newton's three Laws of Motion, which will be regarded as fundamental axioms, and shall first inquire how far they will carry us; and we shall find that when we consider the motion of masses of matter of finite size, Newton's Laws are not sufficient to enable us to determine the motion unless the matter is in the ideal state of a frictionless or perfect fluid. In all other cases an additional hypothesis is necessary.

To clear the ground, it may be well to point out that the parallelogram of forces is a direct consequence of Newton's second law, and that the parallelogram of couples is a consequence of the parallelogram of forces; whence all the propositions relating to the composition of forces and couples, including the one which enables any system of forces and couples to be compounded into a wrench, are deductions from the second law.

There are two methods of investigating the motion of a mass of matter of finite size. In the first place, we may suppose the matter to consist of very small discrete masses under the influence of molecular forces, together with bodily forces, such as gravity and the like; in the second place, we may regard the mass as a continuous one which may be subdivided into small differential elements of volume. We shall consider the subject from the first point of view.

Let $m_1$, $m_2$ be the masses of any two elements $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$, their co-ordinates referred to *fixed* axes; let $R_{12}$ be the molecular force of $m_2$ on $m_1$, $R_{21}$ of $m_1$ on $m_2$; also let $F_1$ be the bodily force acting on $m_1$.

By Newton's second law, the forces $R_{12}$, $F_1$ may be resolved into components $f_{12}$, $g_{12}$, $h_{12}$, and $X_1$, $Y_1$, $Z_1$ parallel to the axes; also by the same law, the equations of motion of the elements are

$$\left.\begin{aligned} m_1\ddot{x}_1 &= X_1 + f_{12} + f_{13} + \dots \\ m_2\ddot{x}_2 &= X_2 + f_{21} + f_{23} + \dots \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$\left.\begin{aligned} m_1\ddot{y}_1 &= Y_1 + g_{12} + g_{13} + \dots \\ m_2\ddot{y}_2 &= Y_2 + g_{21} + g_{23} + \dots \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

It follows from Newton's third law that the molecular force of $m_2$ on $m_1$ is equal in magnitude and opposite in direction to that of $m_1$ on $m_2$; whence $f_{12} = -f_{21}$, &c. Accordingly, if we add the system of equations (1) it follows that all the molecular forces disappear, and we obtain

$$\Sigma(m\ddot{x}) = \Sigma(X) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) is the analytical expression for the principle of linear momentum, and from the above investigation it follows that this principle is a direct consequence of the second and third laws.

Since the equations of motion of a perfect fluid can be deduced from the above principle, it follows that the whole theory of the hydrodynamics of frictionless fluids depends solely on Newton's second and third laws.

We must next consider the principle of angular momentum. From (1) and (2) we obtain

$$\Sigma m(x\ddot{y} - y\ddot{x}) = \Sigma(xY - yX) + g_{12}x_1 + g_{21}x_2 - f_{12}y_1 - f_{21}y_2 + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By the third law the last four terms may be written

$$f_{12}(y_2 - y_1) - g_{12}(x_2 - x_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which represents one of the components of the couple due to the mutual action of $m_1$ and $m_2$.

Now Newton's third law states that the action of $m_1$ on $m_2$ is equal and opposite to that of $m_2$ on $m_1$; but it makes no assertion to the effect that the mutual action consists of a force *acting along the line joining them*. An assumption of this kind would limit the generality of the law, and would be one of a somewhat doubtful character, for in viscous systems there are grounds for thinking that this mutual action may consist in part

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of a force perpendicular to the line joining two elements. If in any particular case the mutual force between two elements *did* act along the line joining them, all the expressions of the form (5) would vanish; but the point to which I particularly wish to call attention to is that *the principle of angular momentum cannot be deduced from Newton's laws, but a further hypothesis is necessary.*

We have not yet considered Newton's first law, and if we discard the hypothesis that the mutual action between two elements consists of a force acting along the line joining them, the next step is to inquire whether the first law will assist us. We have already pointed out that all the molecular forces[1] may be compounded into a wrench. Now Newton's first law asserts, in effect, that when the mass is at rest or is moving uniformly in a straight line, this wrench is zero; but the first law is limited to these two cases, and asserts nothing with regard to what happens when the mass possesses *acceleration*, and it is easy to see that when the mass is in either of the states contemplated by Newton's first law, the molecular forces are not the same as when it is in other states. For example, the molecular forces exerted across any section of a pendulum rod, which is oscillating, are not the same as when the rod is held at rest in any of the positions which it assumes during its motion. Newton's first law does not therefore help us. We have, however, proved by Newton's second and third laws that the *force* constituent of the wrench due to molecular forces is zero; and if we assume that the *couple* constituent is also zero, the sum of all such terms as (5) will vanish, and (4) becomes

$$\Sigma m(x\ddot{y} - y\ddot{x}) = \Sigma(x\mathrm{Y} - y\mathrm{X}) \; . \; . \; . \; . \; . \; . \; (6)$$

which is the analytical expression for the principle of angular momentum.

The latter principle may therefore be deduced from Newton's laws with the aid of one or other of the following additional hypotheses, viz.:—

(*a*) *The molecular action between two elements consists of a force acting along the line joining them.* Or,

(*b*) *The resultant couple due to molecular action is zero, whether the mass of matter be at rest or in motion.*

We must lastly consider the principle of energy.

Assuming that the bodily forces have a potential, we easily deduce from (1) and (2)—

$$\tfrac{1}{2}\frac{d}{dt}\Sigma m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{d\mathrm{V}}{dt} = (f_{12} + f_{13} + \ldots)\dot{x}_1$$
$$+ (g_{12} + g_{13} + \ldots)\dot{y}_1 + \ldots + (f_{21} + f_{23} + \ldots)\dot{x}_2 + \ldots \; . \; (7)$$

whence if T denote the kinetic energy, and F denote the right-hand side of (7) the equation becomes

$$\frac{d\mathrm{T}}{dt} + \frac{d\mathrm{V}}{dt} = \mathrm{F}. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

The function F is an unknown quantity which it is impossible to determine without making some special hypothesis respecting the constitution of matter or the law of molecular force. We shall presently show that F is zero when the matter is rigid. In the case of a viscous liquid it can be shown by means of the general equations (which depend upon the principles of linear and angular momenta, combined with a certain hypothesis concerning molecular action) that F is the dissipation function; whilst, in the case of an elastic solid it can similarly be shown that − F represents the time variation of the potential energy due to strain. That F is zero when the matter consists of a rigid body can be proved as follows:—Let $\omega_1$, $\omega_2$, $\omega_3$ be its component angular velocities about the axes; then

$$\dot{x}_2 = \dot{x}_1 - (y_2 - y_1)\omega_3 + (z_2 - z_1)\omega_2$$

and the terms in F depending on the mutual action of $m_1$ and $m_2$ become

$$(f_{12} + f_{21})\dot{x}_1 - f_{21}(y_2 - y_1)\omega_3 + f_{21}(\mathrm{Z}_2 - \mathrm{Z}_1)\omega_2$$
$$+ \text{ similar terms in } g \text{ and } h.$$

This may be written—

$$(f_{12} + f_{21})\dot{x}_1 - \omega_3\{f_{21}(y_2 - y_1) - g_{21}(x_2 - x_1\} - \ldots$$

By the third law $f_{12} + f_{21} = 0$, and also by either of the subsidiary hypotheses (*a*) or (*b*) all the expressions of the form

$$\omega_3\Sigma\{f_{21}(y_2 - y_1) - g_{21}(x_2 - x_1)\}$$

vanish; whence F = 0, and (8) becomes

$$\mathrm{T} + \mathrm{V} = \text{const.},$$

which is the analytical form of the principle of the conservation of mechanical energy.

A. B. Basset.

## The Artificial Formation of the Diamond.

With your kind permission I wish to make a few observation on Prof. Joly's interesting letter in your issue of March 22, in which he concludes from a consideration of the curve representing the expansion of diamond by heat that "the diamond is a form of carbon which has been subjected to high pressure when crystallising," and in which he remarks that these theoretical considerations gave rise to experiments which he only laid aside finally upon hearing of M. Moissan's success. He also says: "M. Moissan has shown that the *added condition* of high pressure has rendered a method previously unsuccessful *now for the first time successful*." (The italics are mine.) Prof. Joly here makes a claim for M. Moissan which I must say, in justice to that gentleman, he has never made for himself.

If Prof. Joly will kindly turn to my paper on the "Artificial Formation of the Diamond," in the *Proceedings* of the Royal Society, No. 204, 1880, he will find that pressure is *not* an "added condition," and that M. Moissan has *not* "rendered a method hitherto unsuccessful for the first time successful."

Further, he will find that his theoretical deductions were practically demonstrated by me fourteen years ago, and I then described experiments showing that carbon, set free by metals under great pressure, was denser and harder the higher the pressure, and my experiments culminated in the preparation of minute quantities of carbon identical in hardness, density, and action on polarised light with natural diamond.

M. Moissan has done no more, but he has devised a process of great ingenuity which does not entail the danger and expense of mine, and which can be repeated with much greater chance of success.

If Prof. Joly will turn to my paper, he will find that, like Moissan, I obtained my diamond in conjunction with a fused mass containing iron, and fused under extreme pressure. Again, in the *Proceedings* of the Royal Society, of June 14, 1888, he will find that the Hon. C. A. Parsons there describes experiments which produced true diamond (thoroughly corroborated since the publication of his paper), and he proved conclusively that pressure is absolutely essential; and as his method is one which lends itself to the treatment of carbon in quantity at the necessary temperature and at any pressure, it is of much greater practical importance than Moissan's method.

I have the highest admiration of Moissan's work, not only with his electric furnace, but in the several fields in which he has distinguished himself; and as he has courteously admitted that I was the first to define the conditions and actually prepare artificial diamond, I feel constrained to point out that both Parsons and I had fully enunciated the conditions which Prof. Joly attributes to Moissan.

I quite agree with Prof. Joly's concluding paragraph, and my researches led me about a year ago to the construction of an apparatus capable of producing pressures up to forty tons on the square inch, in which I have good hopes of being able to melt carbon in quantity and produce diamond by fusion instead of solution.

Carbon, at ordinary pressures, passes directly from the solid to the gaseous state, and only under enormous pressure can it be made to pass through the liquid state.

It would be premature for me to say more at present, but I hope you will kindly allow me to put these facts on record, as they seem to have been forgotten even by those who are working in the field.

J. B. Hannay.

Cove Castle, Loch Long, N.B., March 26.

I was in hopes that the limited claim for priority which I made for M. Moissan would have secured me from controversy as to the claims of previous workers. Although well acquainted with the paper to which Mr. Hannay refers me, I had never derived from it the idea that Mr. Hannay attributed his results to the crystallisation of carbon out of a metal.

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[1] It seems hardly necessary to suppose that the mutual action between two elements consists of a *couple* as well as a force; for even in the case of magnetized matter, we may regard each element as the limit of a pair of positive and negative ones which are rigidly connected together, and we may deduce the theorem that the mutual action between two magnetic elements consists of a couple by considering the forces between each pair of simple elements.

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